modal truncation error, etc., the pole allocation process can be a viable preliminary control design tool. In view of the trend of modern spacecraft to carry onboard computers, the implementation of a multivariable controller/observer control subsystem to a complex flexible spacecraft becomes more physically realizable than ever before. The ease with which a set of linear controllers satisfying key performance specifications can be directly obtained will considerably simplify the preliminary control design effort for future spacecraft.

Acknowledgment

The work of this paper was performed when both authors were at RCA Corporation, Astro-Electronics, Princeton, New Jersey. The authors wish to express their deep appreciation to RCA for the continuing support of this work.

References

[1] Tseng, G.T. and Phillips, K.J., "Attitude Stability of a Flexible Dual-Spin Spacecraft With Active Nutation Damping Using Products of Inertia," *The Journal of the Astronautical Sciences*, Vol. XXIV, July-Sept., 1976, pp. 257-272.

[2] Pistiner, J.S., Tseng, G.T., and Muhlfelder, L., "Multiloop Analysis of a Precision-Pointing Spacecraft With Controlled Flexible Appendages," *Journal of Spacecraft and Rockets*, Vol. 12, Oct. 1975, pp. 586-591.

[3] Larson, V. and Likins, P.W., "An Application of Modern Control Theory to An Elastic Spacecraft," *ESA Symposium on Dynamics and Contol of Non-Rigid Space Vehicles*, Frascati-Roma, Italy, May 24-26, 1976.

[4] Tracy, T.G. and Tseng, G.T., "Application of Quadratic Cost Optimal Control Theory to a Complex Flexible Spacecraft." *AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft*, Blacksburg, Va., June 13-15, 1977.

[5] Davison, E.J., "On Pole Assignment in Linear Systems with Incomplete State Feedback," *IEEE Transactions on Automatic Control*, Vol. AC-15, June 1970, pp. 348-351.

[6] Davison, E.J. and Chow, S.G., "An Algorithm for the Assignment of Closed-Loop Poles Using Output Feedback in Large Multivariable Systems," *IEEE Transactions on Automatic Control*, Vol. AC-18, Feb. 1973, pp. 74-75.

[7] Davison, E.J. and Wang, S.H., "On Pole Assignment in Linear Multivariable Systems Using Output Feedback," *IEEE Transactions on Automatic Control*, Vol. AC-20, Aug. 1975, pp. 516-518.

[8] Mahn, Jr., R.H., "Algorithm for and Implementation of Pole Assignment Using Output Feedback in Linear Multivariable Time-Invariant Systems," M.S. Thesis, Massachusetts Institute of Technology, June, 1977.

[9] Likins, P.W., "Dynamics and Control of Flexible Space Vehicles," Jet Propulsion Laboratory, JPL TR 32-1329, Rev. 1, Jan. 15, 1970.

[10] Luenberger, D.G., "An Introduction to Observers," *IEEE Transactions on Automatic Control*, Vol. AC-16, Dec. 1971, pp. 596-602.

Flat Spin Recovery of a Rigid Asymmetric Spacecraft

Donald L. Cronin*
University of Missouri – Rolla, Rolla, Mo.

Introduction

THE motion of any spacecraft having nonzero angular momentum will decay to flat spin – spin about the axis of maximum moment of inertia – in the absence of active attitude control. Techniques for flat-spin recovery, that is, for re-establishing desired spacecraft attitude, have been discussed for the spinner intended to spin about its axis of minimum moment of inertia,[1] for momentum bias spacecraft,[2,3] and for dual-spin spacecraft.[4]

The treatment in Ref. 1 of the recovery strategy employing spin-up thrusters for spinners, is, however, approximate and leads to an expression for the requisite recovery torque that, while suitable for the configuration then of interest, is inappropriate for the general asymmetric spacecraft. In this Note, flat-spin recovery for the spinner is addressed exactly, and a generally valid expression for the requisite spin-up thruster recovery torque is derived.

Analysis

The motion of a rigid asymmetric spacecraft initially executing pure spin about its axis of maximum moment of inertia and influenced by a torque $T(t)$ applied about its axis of minimum moment of inertia is described by

$$A\dot{\omega}_1 + (C-B)\omega_2\omega_3 = T(t) \tag{1a}$$

$$B\dot{\omega}_2 + (A-C)\omega_1\omega_3 = 0 \tag{1b}$$

$$C\dot{\omega}_3 + (B-A)\omega_1\omega_2 = 0 \tag{1c}$$

with initial angular rates

$$\omega_1(0) = \omega_2(0) = 0, \qquad \omega_3(0) = \Omega \tag{2}$$

and with the inequality on A, B, and C, spacecraft moments of inertia about body-fixed axis 1, 2, and 3, respectively

$$C > B > A$$

Manipulation of Eqs. (1b) and (1c) leads to an integral of motion

$$s_2\omega_3^2 + s_3\omega_2^2 = s_2\Omega^2 \tag{3}$$

where

$$s_2 = (C-A)/B, \qquad s_3 = (B-A)/C$$

The form of Eq. (3) suggests natural substitutions

$$\omega_2 = \Omega\sqrt{s_2/s_3}\,\sin\eta, \qquad \omega_3 = \Omega\cos\eta \tag{4}$$

In either Eq. (1b) or (1c), substitutions (4) yield

$$\omega_1 = \dot{\eta}/\sqrt{s_2 s_3} \tag{5}$$

Substitutions (4) and (5) together with

$$\phi = 2\eta \qquad \tau = \sqrt{s_1 s_2}\,\Omega t \tag{6}$$

where

$$s_1 = (C-B)/A$$

transform Eq. (1a) into

$$\phi'' + \sin\phi = \kappa(\tau) \tag{7}$$

where the prime denotes a differentiation with respect to τ and where

$$\kappa = 2T/(s_1 A\Omega^2)\cdot\sqrt{s_3/s_2} \tag{8}$$

Initial conditions associated with Eq. (7) may be developed readily from the initial angular rates (2)

$$\phi'(0) = \phi(0) = 0 \tag{9}$$

Received Jan. 10, 1977; revision received Dec. 5, 1977.

Index category: Spacecraft Dynamics and Control.

*Professor, Department of Mechanical and Aerospace Engineering.

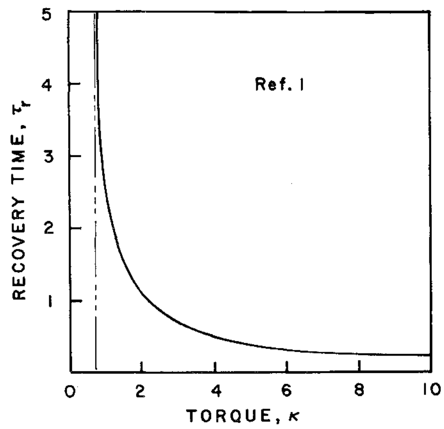

Fig. 1 Nondimensional recovery torque vs nondimensional recovery time (from Ref. 1).

Flat spin recovery of a spacecraft is thus equivalent to the process of driving a simple pendulum from rest at $\phi = 0$.

An integration of Eq. (7) for the case of a constant applied torque gives

$$\tfrac{1}{2}\phi'^2 = \kappa\phi + \cos\phi - 1 \tag{10}$$

for the stated initial conditions.

Pendulum motion not restricted to the neighborhood of $\phi = 0$ and spacecraft motion not restricted to the neighborhood of pure spin about the third axis require values for κ that produce real velocities ϕ' for all ϕ. The smallest such value represents the minimum nondimensional torque required for flat-spin recovery. This value is given in Ref. 1.

$$\kappa > 0.7246 \tag{11}$$

Substitution of the right-hand side of Eq. (8) into inequality (11) leads to the condition on flat spin recovery torque

$$T > 0.3623(C-B)\Omega^2\sqrt{\frac{C(C-A)}{B(B-A)}} \tag{12}$$

Inequality (12) differs from inequality (19) of Ref. 1 by the multiplicative radical. The radical did not appear in Ref. 1 because of the assumption that ω_2 and ω_3 have the same magnitude. The actual magnitudes of these rates are shown here in Eqs. (4). Omission of the multiplicative radical in inequality (12) can lead to a significant error in a recovery-torque determination for spacecraft having significantly different maximum and intermediate moments of inertia.

Figure 3 of Ref. 1 illustrates the time for which a given torque κ must act to produce a successful recovery. This figure is reproduced here as Fig. 1 for completeness. The nondimensional time shown is related to real time by the second of substitutions (6).

After the appropriate recovery impulse has been applied, spacecraft motion will have been transformed into spin about the first axis combined with nonzero rates ω_2 and ω_3. Once this state has been attained, active attitude control, a feature not included in the present model, may be engaged to remove the latter rates in order to produce pure spin about the first axis.

References

[1] Barba, P. M., Furumoto, N., and Leliakov, I. P., "Techniques for Flat-Spin Recovery of Spinning Satellites," AIAA Paper 73-859, Key Biscayne, Florida, Aug. 1973.

[2] Kaplan, M. H. and Patterson, T. C., "Attitude Acquisition Maneuvers for Bias Momentum Satellites," *COMSAT Technical Review*, Vol. 6, Spring 1976, pp. 1-23.

[3] Barba, P. M. and Aubrun, J. N., "Satellite Attitude Acquisition by Momentum Transfer," *AIAA Journal*, Vol. 14, Oct. 1976, pp. 1382-1386.

[4] Gebman, J. R. and Mingori, D. L., "Perturbation Solution for the Flat-Spin Recovery of Dual-Spin Spacecraft," *AIAA Journal*, Vol. 14, July 1976, pp. 859-867.

Analytic Steady-State Accuracy Solutions for Two Common Spacecraft Attitude Estimators

R. L. Farrenkopf*
TRW Defense and Space Systems Group, Redondo Beach, Calif.

Introduction

THIS Note treats the following two attitude estimation problems:

1) The single-axis attitude of a spacecraft is estimated by integrating the output of a rate gyro corrupted by drift rate and zero mean additive white Gaussian noise. The gyro drift rate fails to remain constant, its time derivative also behaving as a zero mean white noise process. Every T seconds, an attitude sensor (e.g., star tracker) measures the spacecraft attitude, and this information is utilized by an optimal estimator (i.e., Kalman filter) to update both the spacecraft attitude and the gyro drift rate. This attitude measurement also is corrupted by zero mean noise. Assuming that the estimator has converged to steady-state operation, what are the standard deviations of the attitude and gyro drift rate estimation errors just prior to, and also just after an attitude update?

2) The single-axis attitude of a spacecraft is measured every T seconds by an attitude sensor (e.g., earth sensor) that is corrupted by zero mean noise. Attitude and attitude rate estimates between samples are provided by a dynamic model of the spacecraft's angular motion, making use of the estimated control and disturbance torques acting upon the spacecraft. These torque estimates are generally imperfect, and, as a result, it is assumed that the true vehicular acceleration differs from its estimate by a zero mean Gaussian white noise process. An optimal estimator compensates these modeling errors by updating both the attitude estimate and its rate subsequent to each attitude measurement sample. Assuming that the estimator has converged to steady-state operation, what are the standard deviations of the attitude and rate estimation errors just prior to a measurement update? Similarly, what are these errors just subsequent to such an update?

It will be shown that the analytic representation of problem 2 is similar to that of problem 1. The latter was considered first by the author in Ref. 1, in which graphical results were presented. Since then, a complete analytic solution has been discovered, and this will be presented in what follows.

Problem Solution

Problem 1

It is assumed that the relatively fixed gyro error sources (e.g., scale factor) can be calibrated by an appropriate

Presented as Paper 74-903 at the AIAA Mechanics and Control of Flight Conference, Anaheim, Calif., Aug. 5-9, 1974; submitted Aug. 9, 1974; revision received Jan. 4, 1978.

Index categories: Spacecraft Systems; Spacecraft Dynamics and Control; Spacecraft Navigation, Guidance, and Flight-Path Control.

*Senior Staff Engineer, Systems Engineering.